

**Event-by-event fluctuations in collective quantities**

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1 Cyclotron Rd., Berkeley, CA 94720***ABSTRACT**

We discuss an event-by-event fluctuation analysis of particle production in heavy ion collisions. We compare different approaches to the evaluation of the event-by-event dynamical fluctuations in quantities defined on groups of particles, such quantities as mean transverse momentum, transverse momentum spectra slope, strength of anisotropic flow, etc.. The direct computation of the dynamical fluctuations and the sub-event method are discussed in more detail. We also show how the fluctuation in different variables can be related to each other.

**I. INTRODUCTION: STATISTICAL, DYNAMICAL, AND EVENT-BY-EVENT FLUCTUATIONS**

Any physical quantity measured in an experiment is subject to fluctuations. In general, these fluctuations depend on the properties of the system and may contain important information about that system. In the context of heavy ion collisions, the system under consideration is a dense and hot fireball consisting of hadronic and/or possibly partonic matter. The obvious challenge is to positively identify the existence of a state of partonic matter early on in the life of the fireball. The study of fluctuations may help in this task

considerably. First of all, fluctuations of a thermal system are directly related to its various susceptibilities [1], which in turn are good indicators for possible phase changes. For example, the extraction of the system heat capacity from temperature fluctuations has recently been proposed in [2–4]. Also, large event-by-event fluctuations may indicate the existence of distinct event-classes, e.g. one with and one without a Quark Gluon Plasma.

Fluctuations have contributions of different nature. First there are ‘trivial’ fluctuations due to a finite number of particles used to define a particular observable in a given event. Examples of such observables are the mean transverse momentum,  $\langle p_t \rangle$ , where the average is taken over all particles in a given event, the strength of anisotropic flow, the ratios of multiplicities of different particle species, etc. Fluctuations due to finite multiplicity we shall call *statistical* fluctuations. Statistical fluctuations can be evaluated by considering the production of all particles as totally independent. All other fluctuations are of dynamical origin and shall be called *dynamical* fluctuations. Dynamical fluctuations can be subdivided into two classes: a) fluctuations which do not change event-by-event (such as two-particle correlations due to Bose-Einstein statistics or due to resonance decays), and b) fluctuations which occur on an event-by-event basis. The last ones we call event-by-event (EbE) fluctuations. Examples of those are fluctuations in the ratio of charged to neutral particle multiplicities due to creation of regions of DCC, or the fluctuations in anisotropic flow due to creations of regions with “unusually” soft/hard equation of state. Also, the occurrence of jets may give rise to event-by event fluctuations e.g. in the high  $p_t$  tail of the transverse momentum distribution.

The purpose of this paper is to present and discuss different approaches to the evaluation of dynamical fluctuations, in particular *EbE fluctuations*. We also address the limitations in extracting observables of physical relevance. Here we concentrate on fluctuations of the mean transverse momentum, since experimental data for these fluctuations are already available [5]. Also, fluctuations of the transverse momentum may be related to fluctuations of the temperature, which in turn may provide important information about the properties of the system under study [1–5].

In section II we present several methods of fluctuation analyses and illustrate them in terms of a simple toy-model. We also discuss how these methods are related to each other

and to approaches already presented in the literature. In the next sections we turn to the specific case of fluctuations of the mean transverse momentum. We shall discuss the relation between fluctuations in  $\langle p_t \rangle$  and the temperature. We finally address the question to what extent the heat capacity of the system and the collision energy or centrality dependence thereof can be extracted from these fluctuations.

## II. EVALUATION OF FLUCTUATIONS. “DIRECT” AND “SUB-EVENT” METHODS

### A. Definitions

In this paper we consider fluctuations in collective quantities, the quantities defined on groups of particles. Such a group could be, for example, particles in some rapidity region. It is useful to start with collective *average* (or intensive) quantities, which in rather general form can be defined as

$$X \equiv \langle x \rangle = \frac{\sum_i x_i}{M}, \quad (1)$$

where  $M$  is the particle multiplicity. The sum is taken over all particles in an event, and  $x_i$  is a variable that is defined for each particle. For example, taking  $x = p_t^2/(2m)$ , where  $p_t$  is the particle transverse momentum and  $m$  is the particle mass, would yield for  $X$  an estimator for the (nonrelativistic) temperature; taking  $x = \cos(2(\phi - \Psi_{RP}))$ , where  $(\phi - \Psi_{RP})$  is the particle azimuthal angle with respect to the reaction plane, would give the strength of elliptic flow,  $v_2$ . We use the same notation,  $\langle \dots \rangle$ , for an average over all particles in an event of a quantity defined on a particle, and also for an average over all events of a quantity defined on an event. Then,  $\langle \langle p_t \rangle \rangle$  would mean the average over all events of  $\langle p_t \rangle$ , the mean values of  $p_t$  derived in each event. For an inclusive mean value (an average over the inclusive single particle distribution) we reserve the notation  $\overline{x}$ . For example, the inclusive mean transverse momentum we denote as  $\overline{p_t}$ , which in general does not necessarily coincide with  $\langle \langle p_t \rangle \rangle$ .

The fluctuations in quantity  $X$  are defined by

$$\sigma_X^2 = \langle X^2 \rangle - \langle X \rangle^2 = \langle \langle x \rangle^2 \rangle - \langle \langle x \rangle \rangle^2 \equiv \sigma_{X,stat}^2 + \sigma_{X,dynam}^2. \quad (2)$$

Note that  $\sigma_{X,dynam}^2$  defined in this way can be negative provided that  $\sigma_{X,stat}^2$  refers to the statistical fluctuations in the totally “uncorrelated” particle production scenario, as defined above. Dynamics (and/or kinematics) can suppress the fluctuations in comparison to the case of the independent particle production. Note, however, that the contribution to  $\sigma_{X,dynam}^2$  due to *event-by-event fluctuations* is always positive.

Within a given event sample all three contributions to  $\sigma_X^2$ , statistical fluctuations, event-by-event fluctuations, and dynamical, not EbE, fluctuations, scale differently with the event multiplicity (see also [6,7]). This property can be used as an additional criteria in the experimental separation of different contributions [8]. Statistical fluctuations scale as  $\sigma_{X,stat}^2 \propto 1/M$ , where  $M$  is the event multiplicity. Event-by-event fluctuations,  $\sigma_{X,EbE}^2$ , do not depend on multiplicity. The non-EbE dynamical fluctuations could have in general two terms, one which does not depend on multiplicity, and the second one, which similar to the statistical fluctuations scales as  $\propto 1/M$ . The part which does not depend on multiplicity is mainly due to Bose-Einstein correlations and two-particle final state interactions. We will argue below that the sub-event method permits to eliminate this part from the total fluctuations. The second part is due to resonance (jets) decays, momentum conservation, etc.. Taking all facts together, it means that, provided the multiplicity independent part of dynamical non-EbE fluctuations is eliminated, e.g. by the sub-event method, the multiplicity independent part of  $\sigma_X^2$  is only of event-by-event origin.

## B. “Direct” method

Let us start with a simple example of a two dimensional nonrelativistic ideal gas in thermal equilibrium at temperature  $\tilde{T}$ . In this case the particle transverse momentum distribution is

$$\frac{dn}{dp_t^2} \propto \exp\left(-\frac{p_t^2}{2m\tilde{T}}\right), \quad (3)$$

and the normalized probability density to find a particle with a given  $x \equiv p_t^2/(2m)$  is

$$\frac{dw}{dx} = \frac{1}{\tilde{T}} \exp\left(\frac{-x}{\tilde{T}}\right). \quad (4)$$

Each event in this example is a random selection of  $M$  particles from a thermal bath. For simplicity we assume that  $M$  is constant<sup>a</sup>.

In order to get an estimate for the temperature of the system one needs to fit the slope of the  $p_t$  distribution. Application of a maximum likelihood method yields the best estimator for  $T$

$$T = \frac{\sum_i x_i}{M} \equiv \langle x \rangle = \left\langle \frac{p_t^2}{2m} \right\rangle, \quad (5)$$

which is just the result of the equipartition theorem in two dimensions ( $\langle E \rangle / M = 2 \cdot (T/2)$ ).

The statistical fluctuations in the quantity  $T$  (the fluctuations due to finite multiplicity  $M$ , under assumption of the independent particle production) can be directly calculated:

$$\sigma_{T,stat}^2 = \langle T^2 \rangle - \langle T \rangle^2 = \left\langle \left( \frac{\sum_i x_i}{M} \right)^2 \right\rangle - \left\langle \frac{\sum_i x_i}{M} \right\rangle^2 = \frac{\sigma_{x,incl}^2}{M}, \quad (6)$$

where  $\sigma_{x,incl}^2 \equiv \overline{(x - \bar{x})^2}$  is the variance of the inclusive distribution in  $x$ . For a thermal distribution (4) one has  $\bar{x} = \tilde{T}$ ,  $\overline{x^2} = 2\tilde{T}^2$ , and  $\sigma_{T,stat}^2 = \tilde{T}^2$ . It follows that

$$\frac{(\Delta T)_{stat}^2}{T^2} \equiv \frac{\sigma_{T,stat}^2}{\tilde{T}^2} = \frac{\sigma_{x,incl}^2}{M\tilde{T}^2} = \frac{1}{M}. \quad (7)$$

For practical applications it is very important to know the accuracy in the calculation of  $\sigma_T$ . The direct calculation of the variance of  $\sigma_T$ ,  $\sigma_{\sigma_T}^2$  is straightforward, but rather lengthy. For a large number of events ( $N_{ev} \gg 1$ ) the answer is simple [11]:

$$\sigma_{\sigma_T}^2 \approx \frac{2\sigma_T^4}{N_{ev}} \implies \sigma_{\sigma_T} \approx \frac{\sigma_T^2}{\sqrt{2N_{ev}}} \implies \sigma_{\sigma_T} \approx \frac{\sigma_T}{\sqrt{2N_{ev}}}. \quad (8)$$

In this paper we consider mostly the case when statistical fluctuations are much larger than the dynamical ones. In this case  $\sigma_T \approx \sigma_{T,stat}$  and

$$\sigma_{\sigma_T} \approx \frac{\sigma_{T,stat}}{\sqrt{2N_{ev}}} = \frac{1}{\sqrt{2N_{ev}M}}. \quad (9)$$

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<sup>a</sup> It is sufficient here to assume that the distribution in  $x$  does not depend on the event multiplicity. If this is not the case, it would mean a known source of event-by-event fluctuations (fluctuations in multiplicity), which in principle should be analyzed separately. See also [9,10] for more detail treatment of the multiplicity fluctuations.

### C. “Sub-event method”

It is simpler to use the sub-event method for the calculation of the EbE fluctuations. Just recollect that we are interested in the fluctuations which affect the entire event. If one subdivides such an event into two subsystems, which we call below sub-events “a” and “b”, the quantities defined on these two sub-events should be correlated.

$$\begin{aligned} \langle (T_a - \langle T_a \rangle) (T_b - \langle T_b \rangle) \rangle &= \langle ((\Delta T_a)_{stat} + (\Delta T)_{dynam}) ((\Delta T_b)_{stat} + (\Delta T)_{dynam}) \rangle \\ &= \sigma_{T,dynam}^2. \end{aligned} \tag{10}$$

Note that in the case of an ideal gas, where the fluctuations are entirely of statistical nature the above correlator would yield zero.

The sub-event method permits one to avoid some problems of the “direct” computation of EbE fluctuations. In particular the problems related to the separation of the EbE fluctuations from other dynamical effects, such as Bose-Einstein correlations (the HBT effect). It is not possible to avoid the HBT correlations in the direct approach and one can only perform a rather complicated estimate of its contribution (see, for example, [5]). In the sub-event method one can define the sub-events on different regions, so that particles from two regions are not correlated (in the HBT sense), and the problem simply disappears. For example, one can define sub-events on rapidity regions separated by 0.1 unit of rapidity. The same trick can be used to get rid of the “two track resolution” problem which is quite serious in many experiments. In addition, using the sub-event method it is also possible to study how the “proton temperature” is correlated with the “pion temperature” and many other interesting questions. Unfortunately, we do not know any simple way of avoiding the correlations due to energy-momentum conservation (see also the discussion of this question in [12,13]).

Another way to look at the correlations using the sub-event method is to compare widths of the distributions in  $(T_a - T_b)$  and in  $(T_a + T_b)$ . While the first distribution is governed mostly by statistical fluctuations, the second one contains dynamical fluctuations as well. The difference in the width of the distributions would yield the dynamical fluctuations (see the calculations within the toy model below in this section).

## D. Relations to other methods

The function  $\Phi_x$  is frequently used in the literature [14–17,5] for the event-by-event fluctuations study. It is defined as

$$\Phi_x = \sqrt{\langle Z^2 \rangle / \langle M \rangle} - \sqrt{z^2}, \quad (11)$$

where

$$Z = \sum_i z_i, \quad z_i = x - \bar{x}, \quad (12)$$

and  $x$  is the quantity under study, for example, the transverse momentum. In order to compare  $\Phi_x$  and  $\sigma_{\langle x \rangle, dynam}^2$  results, we first derive a useful formula. We start with the definition of  $\Phi_x$ , given by Eq. (11). Multiplying both sides of the equation by  $(\sqrt{\langle Z^2 \rangle / \langle M \rangle} + \sqrt{z^2})$ , and taking into account that  $\Phi_x$  is the difference between two almost equal quantities,  $(\sqrt{\langle Z^2 \rangle / \langle M \rangle} \approx \sqrt{z^2} \equiv \sigma_{x, incl})$  one gets

$$\begin{aligned} 2\Phi_x \sigma_{x, incl} &\approx \Phi_x (\sqrt{\langle Z^2 \rangle / \langle M \rangle} + \sigma_{x, incl}) \\ &= (\sqrt{\langle Z^2 \rangle / \langle M \rangle} - \sigma_{x, incl}) (\sqrt{\langle Z^2 \rangle / \langle M \rangle} + \sigma_{x, incl}) \\ &= \langle Z^2 \rangle / \langle M \rangle - \sigma_{x, incl}^2. \end{aligned} \quad (13)$$

To proceed further we need the assumption that multiplicity is not correlated with the distributions in  $x$ . Under this assumption

$$\begin{aligned} \frac{\langle Z^2 \rangle}{\langle M \rangle} - \sigma_{x, incl}^2 &= \frac{\langle M \rangle \overline{(x - \bar{x})^2} + \langle M(M-1) \rangle \langle (x_i - \bar{x})(x_j - \bar{x}) \rangle}{\langle M \rangle} - \sigma_{x, incl}^2 \\ &= \frac{\langle M(M-1) \rangle}{\langle M \rangle} \langle (x_i - \bar{x})(x_j - \bar{x}) \rangle \\ &\approx \langle M \rangle \sigma_{\langle x \rangle, dynam}^2 \end{aligned} \quad (14)$$

We end up with the formula (see also [18]):

$$\Phi_x \approx \frac{\sigma_{\langle x \rangle, dynam}^2 \langle M \rangle}{2\sigma_{x, incl}}. \quad (15)$$

From this formula one can see both, strong and weak points of the two analyses using  $\Phi_x$  and  $\sigma_{\langle x \rangle, dynam}^2$ . The quantity which is directly related to the underlying physics is  $\sigma_{\langle x \rangle, dynam}^2$ . In this sense it is preferable. On the other hand, if one want to compare different systems

in order to see if the underlying physics is the same, and events (systems) differ only by the total multiplicity, then one has to take into account that the correlations scale inversely proportional to the event multiplicity. In this sense the multiplication of  $\sigma_{\langle x \rangle, dynam}^2$  by (in this case, observed) multiplicity allows one to check if the physics is changing. This is the advantage of the  $\Phi_x$  approach (as well as any other approach dealing with the quantity proportional to  $\sigma_{\langle x \rangle, dynam}^2 \langle M \rangle$ ). But one should be careful when comparing  $\Phi_x$  measured by different experiments, and even by the same experiment but under different conditions and/or analysis cuts.  $\Phi_x$  is scaled by the *observed* multiplicity. It means that even for the same event sample it would depend, for example, on the track selection cuts.

It is clear from the definition (1) that correlations between the average collective quantities ( $\langle X_a X_b \rangle$ ) and the corresponding fluctuations (in other words, autocorrelations,  $\langle X_a X_a \rangle$ ) can be rewritten using the usual two-particle correlations (the same, as, for example, the second factorial moment used in the study of intermittency, or the discussed above quantity  $\Phi_x$ ). In this sense the correlations in collective variables provide no additional information compared to the two-particle correlations. Details and subtleties of the relation between the two particle correlations and the even-by-event observables have been discussed recently in [9,10].

It should be noted, on the other hand, that it can be much more convenient to work with collective variables. The “signal-to-background” ratio, i.e.  $\sigma_{dynam}/\sigma_{stat}$ , in these variables generally grows as  $\sqrt{M}$ , where  $M$  is the multiplicity. The reason for this is that fluctuations in “background” distribution scale as  $\sqrt{M}$  while the “signal” (strength of flow, change in  $p_t$ , ...) would depend linearly on multiplicity. A good “signal-to-background” ratio can be very important in order to select “unusual” events, i.e. the events with particular strong/weak signal (temperature, strength of flow, etc.). Another advantage of using the quantities defined on a group of particles is a practical one related to computing time. The computation of the two particle correlation function in the traditional way using events with multiplicity of a few hundred or even a few thousand particles does require a lot of computing time.



### E. A toy model

Let us conclude this section by employing a toy Monte-Carlo event generator in order to illustrate how the above discussed formulae work. In this toy model we generate a few sets of 4000 events each; all events are of the same multiplicity  $M = 1000$ . The different sets are generated for different event-by-event fluctuations in temperature, which is distributed in accordance with

$$T = \tilde{T}(1 + \delta(r - 0.5)), \quad (16)$$

where  $r$  is a random number in  $[0,1]$ , and  $\delta$  is a parameter responsible for the strength of the fluctuations. The transverse momentum of each particle is generated in accordance to the distribution

$$\frac{dn}{dp_x dp_y} \propto \exp\left(-\frac{p_x^2 + p_y^2}{2mT}\right). \quad (17)$$

Using the data generated for  $\tilde{T} = 0.05$  GeV and  $\delta = 0.03$  and  $\delta = 0.1$  (the first one is close to the limit of our sensitivity to the dynamical fluctuations for such a data set) we calculate the dynamical fluctuations (in accordance to Eq. (2)) for each group of 500 events. The results are presented in Fig. 1 together with a fit to a constant. The fit values should be compared to the input values of  $\sigma_{dynam}^2/\tilde{T}^2 = \delta^2/12 = (0.03)^2/12 = 0.75 \cdot 10^{-4}$ , and  $(0.1)^2/12 = 0.833 \cdot 10^{-3}$  respectively. A good agreement between the input and the reconstructed values is observed. It is remarkable that the method is sensitive to fluctuations which one would not expect judging only from the single particle spectrum. The distribution in  $p_t^2$  for the case of  $\delta = 0.1$  is presented in Fig. 2 together with an exponential fit. Not only is no deviation from an exponential distribution visible, but the fit quality is very good,  $\chi^2/n.d.f. = 57/72$ .

The next figures, Figures 3–5, are for illustration of the sub-event method. Fig. 3 shows the correlation between temperatures measured in two sub-events. Already from the scatter plot one can see that the two quantities are correlated, which is the consequence of the introduced event-by-event fluctuations. The profile plot, which shows the average temperature of the subevent “b” as a function of the temperature observed in the sub-event “a”, looks even more convincing. One can see that the temperature values reconstructed on two different

subevents are closely correlated. Such an observation unambiguously indicates a presence of dynamical correlations in the data.

Another way to study if the temperature values are correlated is to look at the distributions in  $(T_a - T_b)$  and  $(T_a + T_b)$ , as discussed above. These distributions are presented in Fig. 4. One can see that the distribution in  $(T_a - T_b)$ , containing only statistical fluctuations, is significantly narrower than the distribution in  $(T_a + T_b)$ , which has both statistical and dynamical fluctuations. Using just the RMS values from the plots, one can estimate the dynamical fluctuations as  $\sigma_{T,dynam}^2 = ((4.33)^2 - (3.21)^2) \cdot 10^{-6}/4 = 0.211 \cdot 10^{-5} \text{ GeV}^2$ .

Quantitative analysis of the dynamical fluctuations using Eq. (10) is presented in Fig. 5. The observed strength of the correlation  $\sigma_{T,dynam}^2 = (0.205 \pm 0.011) \cdot 10^{-5} \text{ GeV}^2$  should be compared with the input value of  $\sigma_{T,dynam}^2 = (\tilde{T}\delta)^2/12 = (0.05 \cdot 0.1)^2/12 = 0.208 \cdot 10^{-5} \text{ GeV}^2$ .

### III. CORRELATIONS BETWEEN DIFFERENT COLLECTIVE VARIABLES. RELATIONS BETWEEN FLUCTUATIONS

Often the fluctuations in different variables are tightly connected with each other. For example, let us consider fluctuations in the mean transverse momentum  $\langle p_t \rangle$  and fluctuations in the effective temperature (more precisely, in the slope parameter of the transverse momentum distribution)  $T$ . We assume that  $\langle p_t \rangle$  is uniquely defined by this parameter. Then one can write  $\langle\langle p_t \rangle\rangle = F(\langle T \rangle)$ . Assuming that the fluctuations are of Gaussian nature, arguments from the theory of error propagation give:

$$\sigma_{\langle p_t \rangle, dynam} = |F'(\langle T \rangle)| \sigma_{T, dynam} \implies \frac{\sigma_{\langle p_t \rangle, dynam}}{\langle\langle p_t \rangle\rangle} = \left| \frac{F'(\langle T \rangle)}{F(\langle T \rangle)} \right| \sigma_{T, dynam}. \quad (18)$$

In reality, the  $p_t$  spectra of most particles lie in between two limiting cases  $\langle\langle p_t \rangle\rangle \propto \sqrt{\langle T \rangle}$  (nonrelativistic ideal gas) and  $\langle\langle p_t \rangle\rangle \propto \langle T \rangle$  (ultrarelativistic ideal gas). It follows then that

$$\frac{\sigma_{\langle p_t \rangle, dynam}}{\langle\langle p_t \rangle\rangle} = (0.5 \div 1) \frac{\sigma_T}{\langle T \rangle}. \quad (19)$$

One can apply this relation to recent measurements [5]. In this paper the limits on EbE fluctuation of  $\langle p_t \rangle$  was established as  $\sigma_{\langle p_t \rangle}/\langle\langle p_t \rangle\rangle < 0.01$ . According to our conclusion it means that  $\sigma_T/\langle T \rangle < 0.02$  (conservative estimate). Note that the mean multiplicity used

in this experiment is of the order of  $\langle M \rangle \approx 250$  and the statistical fluctuations in the temperature are of the order of  $\sigma_{T,stat}/\langle T \rangle \approx 1/\sqrt{\langle M \rangle} \approx 0.07$ .

The relation between effective temperature and mean transverse momentum becomes less transparent if at the time of thermal freeze-out sizeable energy/momentum dependent mean field potentials are present. This could be due to mass changes as proposed in the context of chiral symmetry restoration or simply due to long range interactions among the particles. In this case the relation between transverse momentum and temperature,  $F(T)$ , depends on the detailed structure of the mean field forces at play.

#### IV. CAN WE REALLY MEASURE $C_V$ USING $P_T$ SPECTRA?

It has been proposed in [2–4] to measure temperature fluctuations in order to access the heat capacity of the system

$$\left(\frac{\Delta T}{T}\right)^2 \equiv \frac{\sigma_T}{\langle T \rangle} = \frac{1}{C_V}. \quad (20)$$

Such measurements, if possible, can provide very important information about the equation of state, and can be used to detect the phase transitions where the heat capacity could undergo very rapid change. The possibility to get such information becomes one of the major attractions of event-by-event physics. It was assumed in [2,3] that the temperature fluctuations can be evaluated using an event-by-event analysis of the transverse momentum spectra. In this section we question this particular possibility. Our conclusion is that 1) the required temperature fluctuations *cannot* be measured using the information on only particle transverse momentum, and 2) even if the transverse spectra slope fluctuations are sensitive to phase transition, such a relation is more complicated than suggested by Eq. (20).

Our arguments are based on the following observations. Let us consider a two dimensional ideal gas at temperature  $\tilde{T}$ . We would like to use  $M$  particles to define the temperature by measuring  $p_t$  spectra. For simplicity,  $M$  is fixed. An estimate of the temperature would be

$$T = \frac{\sum(p_x^2 + p_y^2)/(2m)}{M}, \quad \langle T \rangle = \tilde{T}. \quad (21)$$

The event-by-event fluctuations in  $T$  can be easily estimated. They are

$$\left(\frac{\sigma_T}{\langle T \rangle}\right)^2 = \left(\frac{\sigma_{T,stat}}{\langle T \rangle}\right)^2 = \frac{1}{M} = \frac{1}{C_V}, \quad (22)$$

taking into account that the heat capacity of a system of  $M$  particles of a two dimensional ideal gas is  $C_V = 2 \cdot (M/2) = M$ . This formula coincides with Eq. (20). Now let us take a three dimensional ideal gas, but use only two components of the particle momentum ( $p_x$  and  $p_y$ ) for an estimate of the temperature. It is obvious that the fluctuations in  $T$  quantitatively do not change compared to the two dimensional case, but now they clearly do not provide us with the knowledge of the heat capacity. The heat capacity has changed to  $C_V = 3/2 M$ . One can continue with such arguments adding to the consideration internal degrees of freedom: the observed fluctuations remain the same while the heat capacity continues to change. Thus, our conclusion on the possibility to access the system heat capacity by measuring the fluctuations in transverse momentum slopes are rather pessimistic. However, if the fraction of the heat capacity that actually is being measured remains constant, one could still hope to see rapid changes in that quantity as the system goes through a phase transition. So, it is definitely interesting to measure an excitation function of the mean transverse momentum fluctuations.

## V. SUMMARY

We have studied event-by-event fluctuations with the direct method and have introduced a new way to determine fluctuations with the sub-event method. A suitable choice of sub-events and the possible combination of particles within a sub-event or between sub-events trivially allows to exclude some dynamical correlations like the HBT correlations or experimental effects like two particle resolution effects.

The relationship to the  $\Phi_x$  variable has been discussed. The fact that correlations between different collective quantities and their fluctuations can be formulated in terms of two-particle correlations has also been discussed in other papers [9]. The importance of the signal-to-background ratio has been pointed out and the fact that large multiplicity detectors help to increase this ratio.

We have applied the methods developed to a toy model and find that fluctuations can be determined with very high sensitivity.

It has been proposed to measure the heat capacity of a system by studying the dynamical temperature fluctuations. We have shown, that the heat capacity cannot be measured from

the temperature fluctuations. However, it cannot be excluded that by carefully measuring an excitation function and the related fluctuations a possible phase transition would manifest itself in increased fluctuations in a (narrow) energy region.

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## FIGURE CAPTIONS

1. Reconstructed dynamical fluctuations for  $\delta = 0.03$  (left panel ) and  $\delta = 0.1$  (right panel).
2. Particle distribution in  $p_t^2$  together with an exponential fit.
3. Sub-event method. Correlations between  $T_a$  and  $T_b$ . Scatter (left panel) and profile (right panel) plots.
4. Sub-event method. Distribution in  $T_a - T_b$  (left panel) and  $T_a + T_b$  (right panel).
5. Sub-event method.  $\langle (T_a - \langle T_a \rangle)(T_b - \langle T_b \rangle) \rangle \equiv \sigma_{T,dynam}^2$  calculated on the 500 event subsamples for  $\delta = 0.1$ .

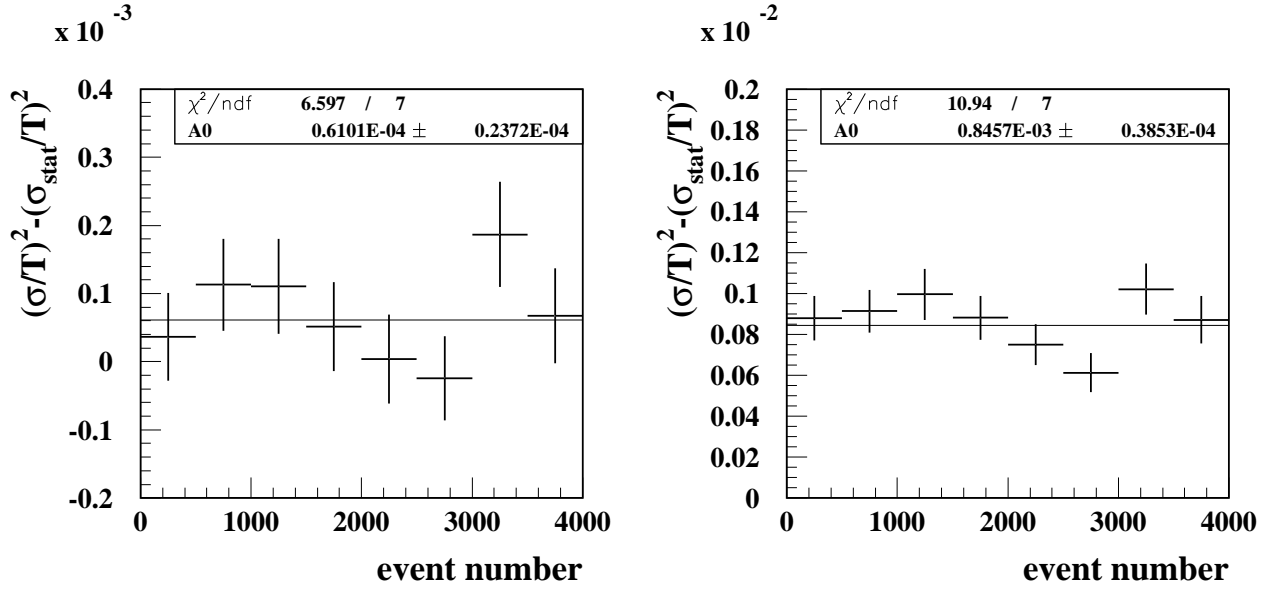


FIG. 1. Reconstructed dynamical fluctuations for  $\delta = 0.03$  (left panel ) and  $\delta = 0.1$  (right panel).

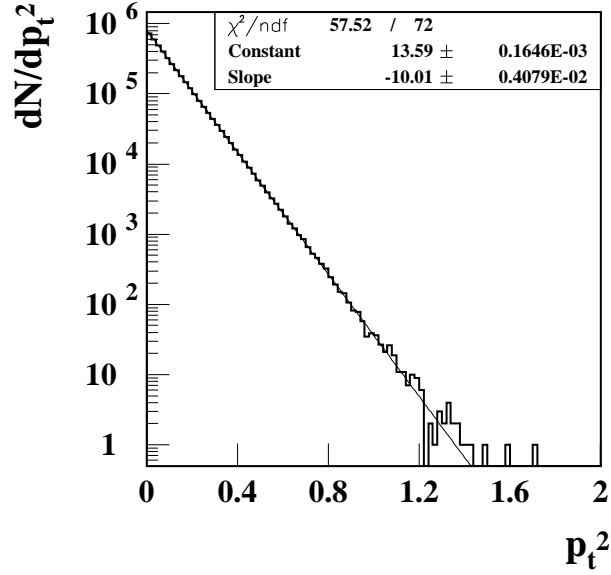


FIG. 2. Particle distribution in  $p_t^2$  together with an exponential fit.



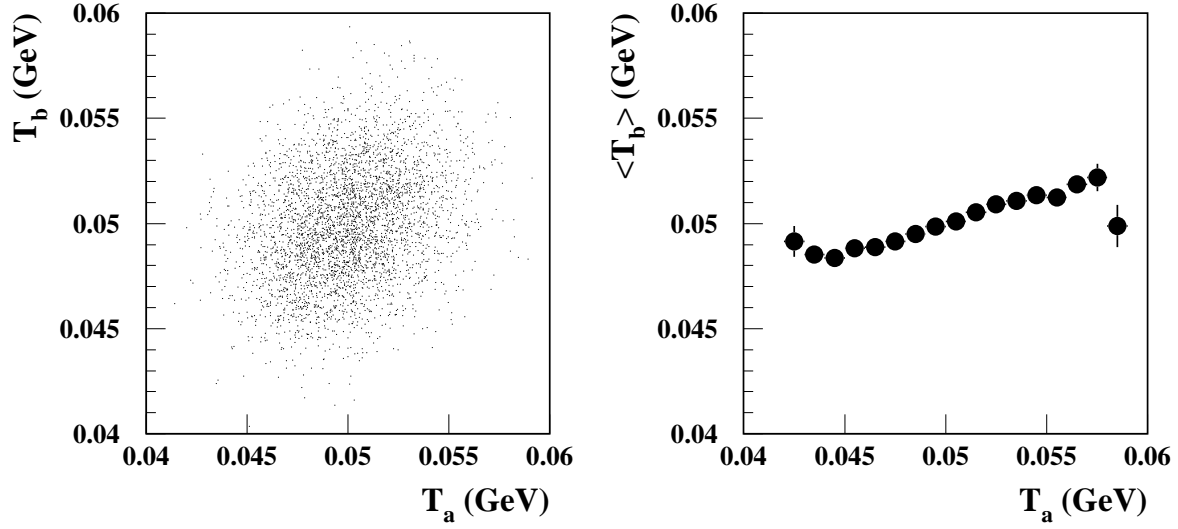


FIG. 3. Sub-event method. Correlations between  $T_a$  and  $T_b$ . Scatter (left panel) and profile (right panel) plots.

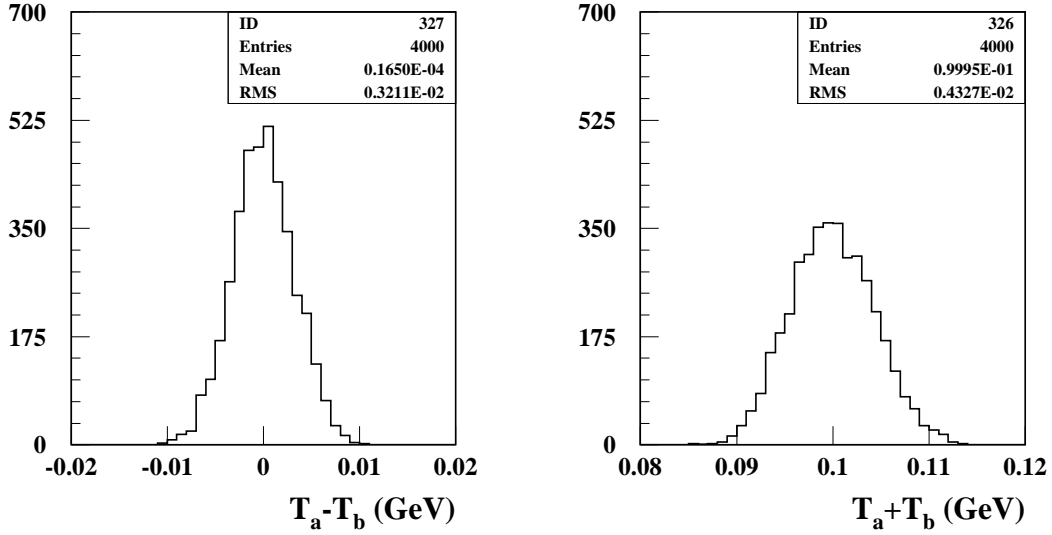


FIG. 4. Sub-event method. Distribution in  $T_a - T_b$  (left panel) and  $T_a + T_b$  (right panel).

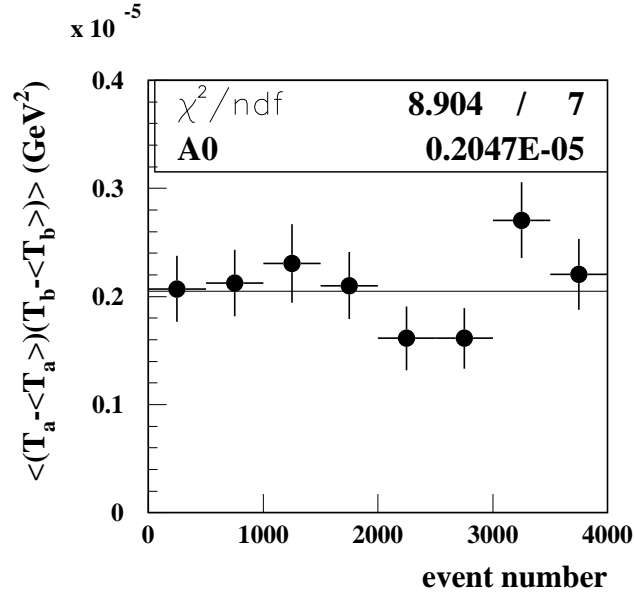


FIG. 5. Sub-event method.  $\langle (T_a - \langle T_a \rangle)(T_b - \langle T_b \rangle) \rangle \equiv \sigma_{T,dynam}^2$  calculated on the 500 event subsamples for  $\delta = 0.1$ .